

# STEP Examiner's Reports 2017

Mathematics

STEP 9465/9470/9475

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## **STEP III 2017 Examiner's Report**

The total entry was only very slightly smaller than that of 2016, which was a record entry, but was still over 10% more than 2015. No question was attempted by in excess of 90%, although two were very popular and also five others were attempted by 60% or more. No question was generally avoided with even the least popular one attracting more than 10% of the entry. Less than 10% of candidates attempted more than 7 questions, and, apart from 18 exceptions, those doing so did not achieve very good totals and seemed to be 'casting around' to find things they could do: the 18 exceptions were very strong candidates who were generally achieving close to full marks on all the questions they attempted. The general trend was that those with six attempts fared better than those with more than six.

#### Question 1

The most popular question on the paper, attempted by about 84% of the candidates, it was also the most successfully answered with an average mark of about 12/20. The first result was generally well answered with a few candidates attempting to use induction, and then proving the result directly. The summations were usually done well, though often lacked explanation. Usually, the inequalities were not well argued, there was poor layout, and no mention of positivity. Those who spotted the link with part (i) did well in general summing the inequalities, though there were some problems with the indices.

# Question 2

This was the least popular pure question being attempted by only just over a quarter of candidates, and was the least successful of all the questions scoring 5/20. Most candidates gave up after part (i), and some made much more of this first result, not being very succinct. Most could write down SR without difficulty, but then did not spot an easy way to move beyond this. The standard of algebra displayed was in general poor, in particular moving between complex and trigonometric forms.

# **Question 3**

The second most popular question at just over 70%, the success rate was about half marks in common with a number of other questions, with the majority earning either 16 and above, or 4 and below. A common mistake was omitting the minus sign in the first step to obtain A which resulted in candidates being unable to progress further. If the cubic equation was correctly found, then candidates tended to score all the marks as far as part (iii). A few candidates obtaining the correct results in (iii) then stated that the answers could not be complex, which was, of course, false.

#### **Question 4**

Three fifths of the candidates attempted question 4 with a marginally better success rate than question 3. A significant proportion of candidates struggled with changing base for part (i), but almost all completed (ii) successfully. A common strategy for part (iii) was to use the result of part (i) but very few remembered to check for b=1. There were very few successful attempts for part (iv); many tried integration by parts, but rarely successfully.

#### **Question 5**

Very slightly more popular than question 4, the marks scored were on average 1 less per attempt. Most found  $\frac{\mathrm{d}y}{\mathrm{d}x}$  successfully, though a significant minority swapped x and y. In this case, they could still obtain the displayed equation successfully, but in both categories, there were frequent sign errors when differentiating trigonometric functions. Most then attempted using the displayed result to find  $f(\theta)$ , either by separating variables or using an integrating factor and got as far as  $f(\theta) = \left(\frac{k\cos^2\theta}{1+\sin\theta}\right)$  but then more than half got stuck. Most plotted the two given curves relatively correctly, but then a substantial number used guesswork having not previously obtained C correctly.

## **Question 6**

The second most popular question attempted by four fifths of the candidates; the success rate was only very slightly less than that of question 4. As every part of the question required obtaining a given result, it had to be marked strictly on how well things were presented. There were surprising problems with changing the variables in the first part, as often candidates did not clearly understand dummy variables, and others integrated with respect to constants. In spite of the ban on the use of trigonometric functions, some still tried to use the tangent function. The two results in (iii), especially the second, were testing but were found very hard, and previous inapplicable results were used, ignoring the conditions given as inequalities.

#### **Question 7**

With popularity between that of questions 4 and 5, the mean score was about 8/20, making it one of the least successful pure questions. Most candidates attempting this question did the stem correctly and then scored about half the marks on (i) before stopping, either due to mistakes in the gradient computation or commonly not identifying the  $(1+t^2)^2$  in the constant term of the line equation. A common slip was to differentiate  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 

implicitly as  $\frac{2x}{a^2} + \frac{2y\frac{\mathrm{d}y}{\mathrm{d}x}}{b^2} = 1$ . The geometric interpretation in (i) was frequently omitted, and there were numerous and varied incorrect suggestions for the case  $X^2 = a^2$ . Few continued to (ii), though it should be observed that some courted disaster by labelling the coefficients of the quadratic in (i) as a, b, and c.

#### **Question 8**

This was attempted as many times as question 5, but the success rate was about halfway between that of questions 5 and 7. Many attempts were made using induction which wasted a lot of time. Otherwise, in general, the stem and part (i) were well solved, but many could not spot the method to proceed with part (ii).

# **Question 9**

The most popular of the applied questions, there were a handful of attempts more for this question than for question 2. However, it was the least successfully attempted applied question with about one third marks scored. Common errors were to assume constant acceleration which does not apply, or to consider the motion of the centre of mass, but ignoring the normal force at the edge of the table, and the fact that the centre of mass does not lie along the string once motion commences. The two constants of integration for the first result were in fact zero but needed to be shown to be so. In considering the energy of the system, many assumed the speeds of A and B were equal, which they do work out to be, but this could not be known before calculating correctly, and likewise the elastic energy being zero, which again needed to be shown. Numerous attempts resulted in the given correct speed from specious working. Scoring largely occurred in the first section of the solution, though rarely earning all the marks for that first result and then earning little attempting to conserve energy.

## **Question 10**

Attempted by about one eighth of the candidates, the success rate was only marginally better than that for question 9. The first displayed result and the expression for  $\ddot{\theta}$  were generally successfully dealt with by those candidates who knew how to apply moments of inertia. After that point, most mistakes were either algebraic or incorrect signs in the equations derived by resolving forces to obtain acceleration. About half of those that reached the end of the question correctly interpreted the physical meaning of the case  $\ell > 2a$ . However, a common misinterpretation was that the particle would begin to slip at this point.

## **Question 11**

A fifth of the candidates attempted this with just a little less success than that for question 5. Only a minority drew a sketch of the problem; had more done so, some errors might have been precluded. In part (i), a large minority ignored the condition 'initially at rest', a handful gave a negative answer for u, and a few attempted to conserve energy, but that was rare. In the second part, some candidates attempted to just write down the given answer employing conserved momentum with verbal justification. The inequality generally followed if a telescoping argument was used although the care shown in dealing with the strict inequality and the r=n case was poor. Attempts at (iii) were generally sound though some took the projectile speed as u.

#### **Question 12**

The least popular question on the paper, it was still attempted by just over 10% of the entry achieving marks only very marginally less good than for question 3. It was fairly well done overall, though a few were completely confused, so the marks tended to either be very high, very low or about around half marks for some who did not do much on part (ii).

# **Question 13**

A sixth of the candidates tried this, scoring slightly less well than was done on question 11. Almost everyone found V(x) correctly and the required result for E(Y). Similar success was demonstrated finding V(x) in the uniform case. A lot did not then attempt to find the probability density function, but most who spotted it attempting to calculate the cumulative distribution function of Y first and then differentiate could do it. It was encouraging that so many correctly found the range of Y. A variety of methods of integration were used for the final result with varying success.